On break points in Continuum theory
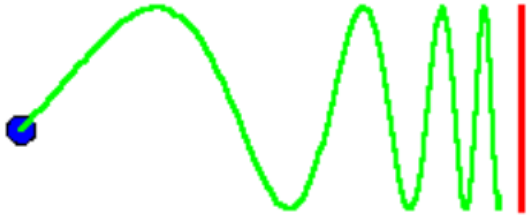
Pavel Pyrih
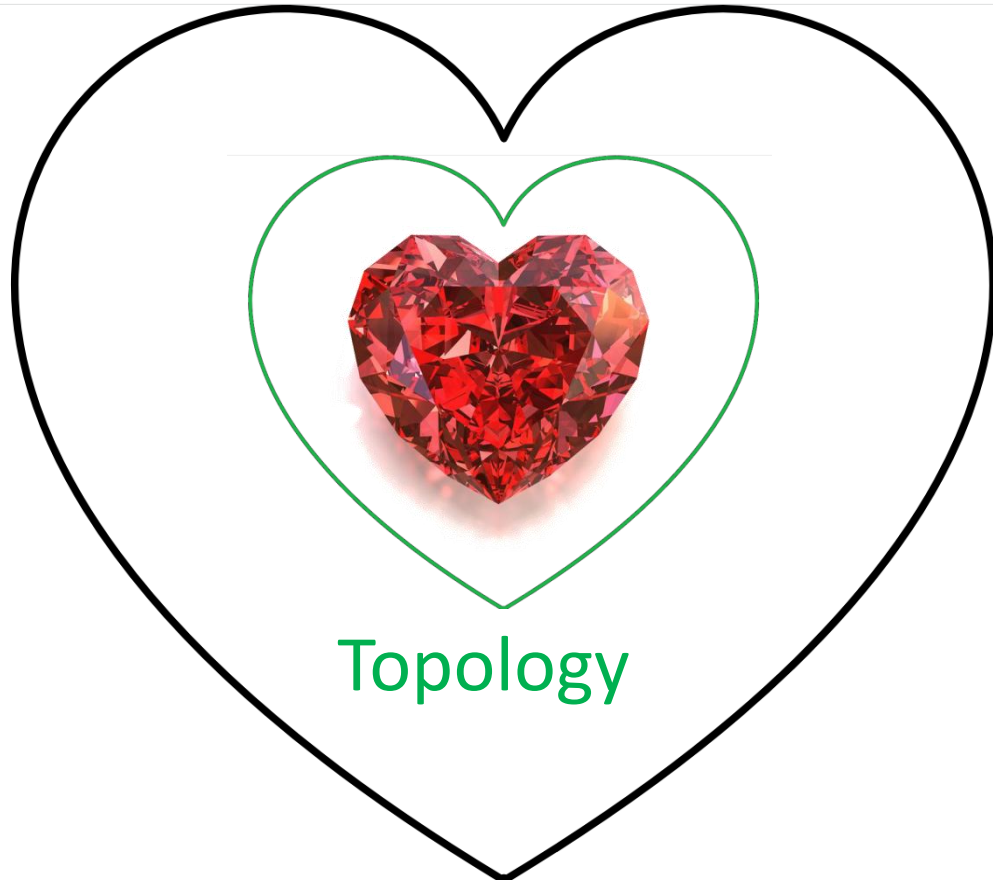
Janusz Jerzy Charatonik & Pawel Krupski & PP

"The way is the goal."

Mahatma Gandhi

Examples in Continuum Theory

Definition
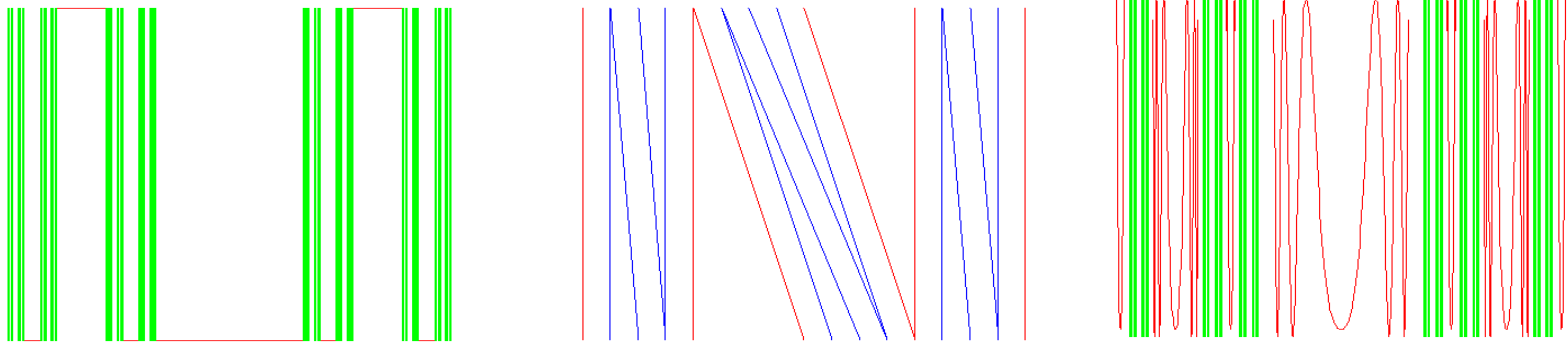
Break point (gem) is an item

(i) written on 1 line

(ii) explained in 5 minutes

Example

Continuum is a compact playground for any small ant.

(Georg Cantor 1883)

Continuum Theory Gem Award

Winners

Discovering a break point (gem) in Continuum Theory.

Award

Single unique composant of any Buckethandle worldwide.

Acknowledgement

Award is given by the Continuum Theory community.

The
Bag End composant
containing the only entry point
was given as the first award to
Georg Cantor
for the definition of a continuum.
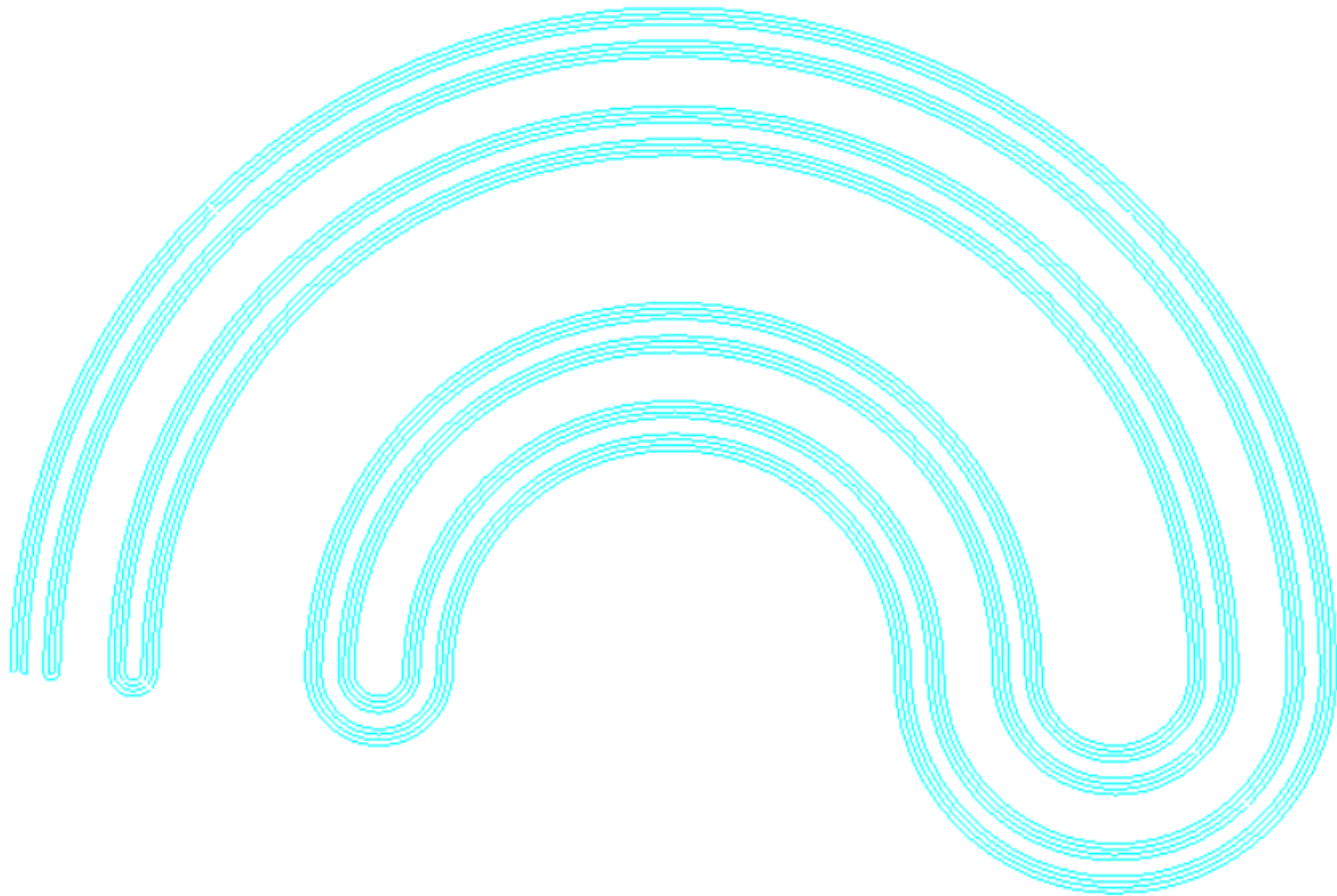
G. Cantor - Continuum Theory Gem Award (1883)

Congratulations to the award winner !!!

Thanks for the gem !!!

1880s

1883	Cantor	G.	**Continuum is a compact playground for any small ant.**
1887	Jordan	C.	Each simple closed planar curve surrounds its inside and outside.

1890s

1890	Peano	G.	Drawing full square with a segment.

1900s

1900	Hilbert	D.	**Hilbert cube containing a copy of any continuum.**
1902	Painleve	P.	A decreasing sequence of continua forms a continuum.

1910s – page 1/2

1910	Brouwer	L.E.J.	Buckethandle as the simplest indecomposable continuum.
1912	Janiszewski	Z.	Arc-less continuum.
1912	Janiszewski	Z.	Buckethandle in an angular shape.
1912	Wada	T.	Lakes of Wada (three disjoint connected open sets of the plane with the same boundary).
1914	Hahn	H.	Locally connected continuum is a continuous image of [0,1].
1914	Mazurkiewicz	S.	Sierpinski carpet contains only ramification points of infinite order.

1910s – page 2/2

1915	Sierpinski	W.	**Half homogeneous Sierpinski carpet containing a copy of every planar curve.**
1915	Sierpinski	W.	Sierpinski triangle with only ramification points.
1916	Moore	R.L.	Chainable continuum.
1918	Sierpinski	W.	Any continuum is sigma connected, not a countable union of disjoint closed sets.
1918	Janiszewski	Z.	Foundation of Fundamenta Mathematicae journal (with W. Sierpinski and S. Mazurkiewicz).

1920s – page 1/3

1920	Antoine	L.	Antoine's necklace (wild embedding of the Cantor set).
1920	Mazurkiewicz	S.	Locally connected continuum is a continuous image of [0,1].
1920	Moore	R.L.	Two non-cut points in any continuum.
1922	Antoine	L.	Antoine's arc (wild embedding of the arc).
1922	**Knaster**	**B.**	**Buckethandle in a round shape.**
1922	**Knaster**	**B.**	**Pseudo-arc, the nondegenerate hereditarily indecomposable continuum.**

1920s – page 2/3

1923	Wazewski	T.	Wazewski universal dendrite containing a copy of any dendrite.
1924	Alexander	J.W.	Alexander horned sphere, a wild embedding of a sphere into space.
1924	Mazurkiewicz	S.	Each locally connected homogeneous plane continuum is a simple closed curve.
1925	Gehman	H.M.	Dendrite with ramification points of order 3 and end points forming the Cantor set.
1926	Menger	K.	Homogeneous Menger sponge containing a copy of every curve.
1927	Kuratowski	K.	A point of irreducibility in a continuum is not contained in two proper subcontinua (a characterization).

1920s – page 3/3

1927	Vietoris	L.	Solenoid (in a sequence folding tube in a previous tube twice).
1929	Roberts	J.H.	Upper semicontinuous decomposition of the plane into nonseparating locally connected subcontinua.

1930s – page 1/3

1930	Mazurkiewicz	S.	A typical continuum is hereditarily indecomposable.
1930	Dantzig	D. von	Homogeneity of a solenoid.
1930	Kurtowski	K.	Planability test for local dendrites (cannot contain two special continua).
1930	Whyburn	G.T.	Whyburn's curve, a continuum every subcontinuum of which separates the plane.
1931	Keller	O.H.	Hilbert cube is homogeneous.
1932	Miller	E.W.	A dendrite not containing its proper copy.

1930s – page 2/3

1932	Waraszkiewicz	Z.	Waraszkiewicz's spirals (an uncountable family of continuously incomparable planar curves).
1934	Borsuk	K.	The dunce hat (Borsuk's tube, acyclic polyhedron which is not the union of two acyclic polyhedra).
1935	Mazurkiewicz	S.	Any compact metric space of dimension 2 contains a nondegenerate indecomposable continuum.
1936	Whyburn	G.T.	Decomposition of 3-space into 3-space using Antoine's arc and points.
1937	Sierpinski	W.	Arc mapped onto Hilbert cube.
1937	Freudenthal	H.	Solenoid as an inverse limit.

1930s – page 3/3

1937	Claytor	S.	Two Claytor continua block planarity of a Peano continuum.

1940s

1944	Sorgenfrey	R.H.	Every nondegenerate unicoherent continuum which is not a triod is irreducible.
1945	Besicovitch	A.S.	A dendrite such that no two of its open sets are homeomorphic.
1947	Borsuk	K.	An example of a simple arc in space whose projection in every plane has interior points.
1948	**Bing**	**R.H.**	**Homogeneity of the pseudo-arc.**
1948	**Moise**	**E.E.**	**Indecomposable planar continuum homeomorphic to each of its nondegenerate subcontinua (the pseudo-arc).**
1948	Jones	F. B.	T function (the complement of T(A) contains the interior points of subcontinua not meeting A).

1950s – page 1/2

1951	Hamilton	O.H.	Each chainable continuum has the fixed-point property.
1951	**Bing**	**R.H.**	**Planar non-chainable circularly chainable hereditarily indecomposable continuum (a pseudo-circle).**
1952	Anderson	R.D.	A continuous decomposition of the plane into nonseparating subcontinua.
1958	Anderson	R.D.	Menger universal curve is homogeneous.
1959	Anderson	R.D.	A plane continuum no two of whose nendegenerate subcontinua are homeomorphic.
1959	Choquet	G.	A plane continuum no two of whose nendegenerate subcontinua are homeomorphic.

1950s – page 2/2

1959	Isbell	J.R.	Chainable continuum is an inverse limit of an inverse sequence of arcs.
1959	**Bing**	**R.H.**	**The circle of pseudo-arcs.**
1959	**Jones**	**F. B.**	**The circle of pseudo-arcs.**

1960s – page 1/2

1961	Lelek	A.	The Lelek fan (a smooth fan with a dense set of endpoints).
1962	Lelek	A.	Weakly chainable continua are exactly continuous images of the pseudo-arc.
1963	Borsuk	K.	Borsuk nonplanar fan.
1964	Henderson	G.	Arc is the only decomposable hereditarily equivalent continuum.
1964	Lelek	A.	Span zero (no two points can change the position on a continuum).
1964	Henderson	G.	The pseudo-arc is an inverse limit using only one binding map.

1960s – page 2/2

1964	Charatonik	J.J.	Unicoherence of locally connected continua is an invariant under confluent mappings.
1965	Effros	E.G.	The group of homeomorphisms acts locally.
1965	**Schori**	**R.M.**	**Universal chainable continuum.**
1967	**Smale**	**S.**	**Buckethandle in a horseshoe shape as the fixed set of a mapping square into a horseshoe.**
1967	Cook	H.	Cook's continuum (a continuum admitting only the identity or a constant mapping onto itself).
1969	Krasinkiewicz	J.	Sierpinski universal plane curve is 1/2-homogeneous.

1970s – page 1/4

1970	Rogers	J.T.	A pseudo-solenoid is a common model for circle-like continua.
1970	**Rogers**	**J.T.**	**The pseudo-circle is not homogeneous.**
1970	**Fearnley**	**L.**	**The pseudo-circle is unique.**
1971	Lelek	A.	Houston problem book.
1972	Ingram	W.T.	An atriodic tree-like continuum with positive span.
1972	Bennett	R.B.	Menger universal curve is countable dense homogeneous.

1970s – page 2/4

1972	Charatonik	J.J.	Problem posing (?) challenge over problem solving (exists) and generalisations (for all).
1973	Nadler	Sam B. Jr.	The indecomposability of the dyadic solenoid.
1974	Rosenholtz	I.	Open image of a chainable continuum is chainable.
1975	Ungar	G.S.	2-homogeneous continuum is locally connected.
1977	Hagopian	Ch.L.	Solenoids (homogeneous continua with only arc subcontinua).
1977	**Rogers**	**J.T.**	**Solenoids of pseudo-arcs.**

1970s – page 3/4

1978	Charatonik	J.J.	Arc-like openly homogenous continuum is the pseudo-arc.
1978	Curtis	D.W.	Hyperspaces of closed subsets of Peano continua are Hilbert cubes.
1978	Schori	R.M.	Hyperspaces of closed subsets of Peano continua are Hilbert cubes.
1978	Illanes	A.	Hyperspaces of sets. Book I
1978	Grispolakis	J.	Universal smooth dendroid.
1978	Tymchatyn	E.D.	Universal smooth dendroid.

1970s – page 4/4

1979	Bellamy	D.P.	A nonplanar tree-like continuum which admits a fixed point free mapping.
1979	Ingram	W.T.	Hereditarily indecomposable tree-like continua.

1980s – page 1/3

1980	Oversteegen	Lex G.	A dendroid with a monotone open map onto an arc.
1980	Mill	J. van	A Peano continuum homeomorphic to its own square but not to its countable infinite product.
1980	Iliadis	S.D.	Universal completely regular continuum (each nondegenerate subcontinuum has a nonempty interior).
1982	Oversteegen	Lex	Indecomposable homogeneous planar continuum has span zero.
1982	Tymchatyn	E.D.	Indecomposable homogeneous planar continuum has span zero.
1984	**Charatonik**	**J.J.**	**Example of a smooth dendroid without ordinary points.**

1980s – page 2/3

1984	Nikiel	J.	Gehman dendroid is the only dendrite with its set of end-points homeomorphic to the Cantor set.
1985	Mackowiak	T.	A universal hereditarily indecomposable continuum.
1986	**Kennedy**	**J.**	**Pseudo-circle has uncountably many orbits each of which is the union of uncountably many composants.**
1986	**Rogers**	**J.T.**	**Pseudo-circle has uncountably many orbits each of which is the union of uncountably many composants.**
1988	Mackowiak	T.	A characterization of finitely irreducible continua.

1980s – page 3/3

1989	Charatonik	J.J.	A fan is smooth iff can be embedded into the Cantor fan.
1989	Charatonik	W.J.	A fan is smooth iff can be embedded into the Cantor fan.
1989	Charatonik	W.J.	Lelek fan is the unique smooth fan with a dense set of endpoints.

1990s – page 1/4

1990	Kuperberg	K.	A locally connected homogeneous non-bihomogeneous continuum of dimension 7.
1990	Krupski	P.	Homogeneous tree-like continua are hereditarily indecomposable.
1990	Prajs	J.R.	Homogeneous tree-like continua are hereditarily indecomposable.
1991	Minc	P.	A transitive map on [0,1] whose inverse limit is the pseudo-arc.
1991	Transue	W.R.R.	A transitive map on [0,1] whose inverse limit is the pseudo-arc.
1992	Nadler	Sam B. Jr.	Continuum Theory. An Introduction. Book II

1990s – page 2/4

1993	Nadler	Sam B. Jr.	A continuum separated by each of its nondegenerate proper subcontinua.
1993	Seldomridge	Gary A.	A continuum separated by each of its nondegenerate proper subcontinua.
1993	Bellamy	D.	A tree-like continuum without the fixed-point property.
1993	Minc	P.	An atriodic simple-4-od-like continuum which is not simple-triod-like.
1993	Aarts	J.M.	Hairy arc.
1993	Oversteegen	L.G.	Hairy arc.

1990s – page 3/4

1994	Bandt	Ch.	Non-zero composants of the Buckethandle are homeomorphic.
1995	Kennedy	J.	Buckethandle as an inverse limit of circles using a single bonding map.
1996	Hagopian	Ch. L.	Simply connected plane continua have the fixed-point property.
1999	**Lewis**	**W.**	**An example of a chainable continuum that admits an exactly 2-to-1 mapping onto a continuum.**
1999	Illanes	A.	Hyperspaces. Book III
1999	Nadler	Sam B. Jr.	Hyperspaces. Book III

1990s – page 4/4

1999	Lewis	W.	Pseudo-arc maturity.
1999	Illanes	A.	The Continuum theory hyperspace in Mexico.
1999	Minc	P.	There exists a tree-like weakly chainable continuum without the fixed-point property.

2000s

2000	Prajs	J.R.	A continuous decomposition of the Menger curve into pseudo-arcs.
2002	**Illanes**	**A.**	**Buckethandle admits no mean.**
2003	Minc	P.	Buckethandle is mapped onto a planar dedroid with point-inverses at most of cardinality 3.
2003	Minc	P.	Planar dendroid contains a single point bottleneck.
2005	Macias	S.	Topics on continua. Book IV

2010s – page 1/3

2010	Illanes	A.	The arc is the only chainable continuum admitting a mean.
2010	Villanueva	H.	The arc is the only chainable continuum admitting a mean.
2011	**Hoehn**	**L.C.**	**A non-chainable plane continuum with span zero.**
2012	Ingram	W.T.	Inverse limit of books on inverse limit...
2013	Vejnar	B.	Acrless-arc, the arc-like arc-less continuum with two endpoints.
2013	Hoehn	L.C.	An uncountable collection of copies of a non-chainable tree-like continuum in the plane.

2010s – page 2/3

2014	Sturm	F.	Pseudo-solenoid is not continuously homogeneous.
2015	Mill	J. van	Pseudo-arc is the only continuum with homeomorphically near property.
2015	Banic	I.	Standard universal dendrite of order 3 as an inverse limit of arcs with one binding map.
2015	Martinez-de-la-Vega	V.	Standard universal dendrite of order 3 as an inverse limit of arcs with one binding map.
2016	Prajs	J.R.	Arcwise connected homogeneous continuum is a continuous image of the Cantor fan.

2010s – page 3/3

2016	**Hoehn**	**L.C.**	**Classification of planar homogeneous continua.**
2016	**Oversteegen**	**Lex**	**Classification of planar homogeneous continua.**
2016	Vejnar	B.	Two non-block points in any continuum.

2020s

2020	Hoehn	L.C.	Classification of hereditarily equivalent plane continua.
2020	Oversteegen	Lex	Classification of hereditarily equivalent plane continua.
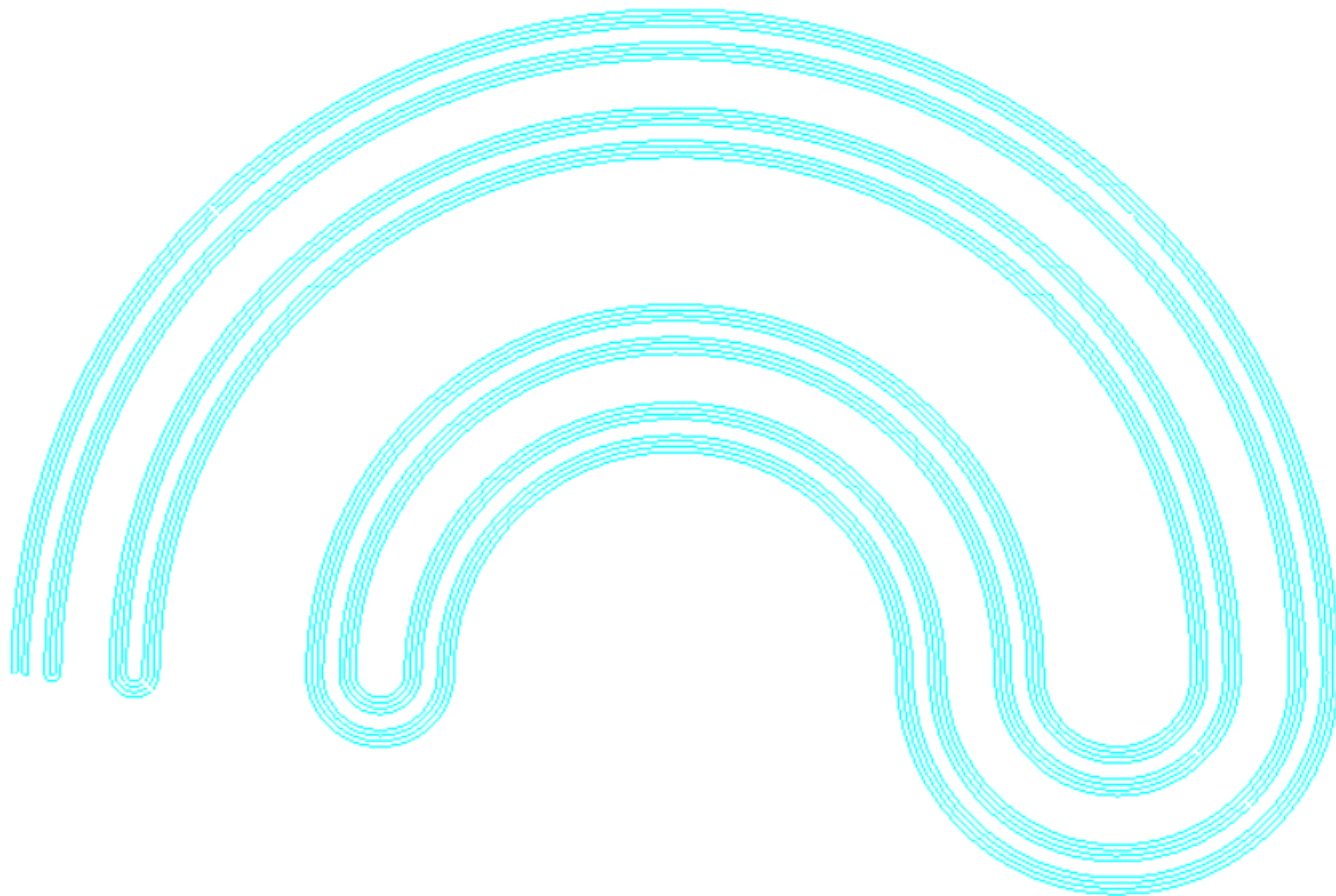
Name - Continuum Theory Gem Award (YEAR)

MORAL

of Continuum Theory Gem Award

- Moral duty to give awards for gems
- History track of events
- History track of people
- …

BENEFITS

of Continuum Theory Gem Award

- Family status

- Unique Buckethandle composant

- Being in a GREAT company

- Looking for the next composant

APPLICATION

of Continuum Theory Gem Award

- Professional status

- Funding, Projects, Grants, Money

- PhD students

- CV
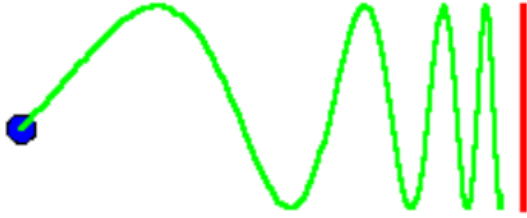
[Examples in Continuum Theory](#)

[Continuum Theory Gem Award](#)

WikipediA – Math Prizes

- 15 international
- 98 America
- 12 Asia
- 78 Europe
- 7 Oceania
- 0 Africa

WikipediA – Math Prizes in topology

- Prize in Topology and Geometry - US
  young female Armenian under 35
- Prize in Geometry - US –
  notable research in geometry or topology
- Geometry prize - Japan –
  geometry, including differential geometry,
  topology, and algebraic geometry

WikipediA – Math Prizes international

- Breakthrough Prize in Mathematics
- Chern Medal
- Continuum Theory Gem Award
- David Hilbert Award
- Felix Chayes Prize
- Fields Medal

…

WikipediA – PUBLISH or nothing

Only already published information can go in WIKI.

Continuum Theory Gem Award list of winners
(1883 - 2020 setup) should be published in

Topology and its Applications
Topology proceedings

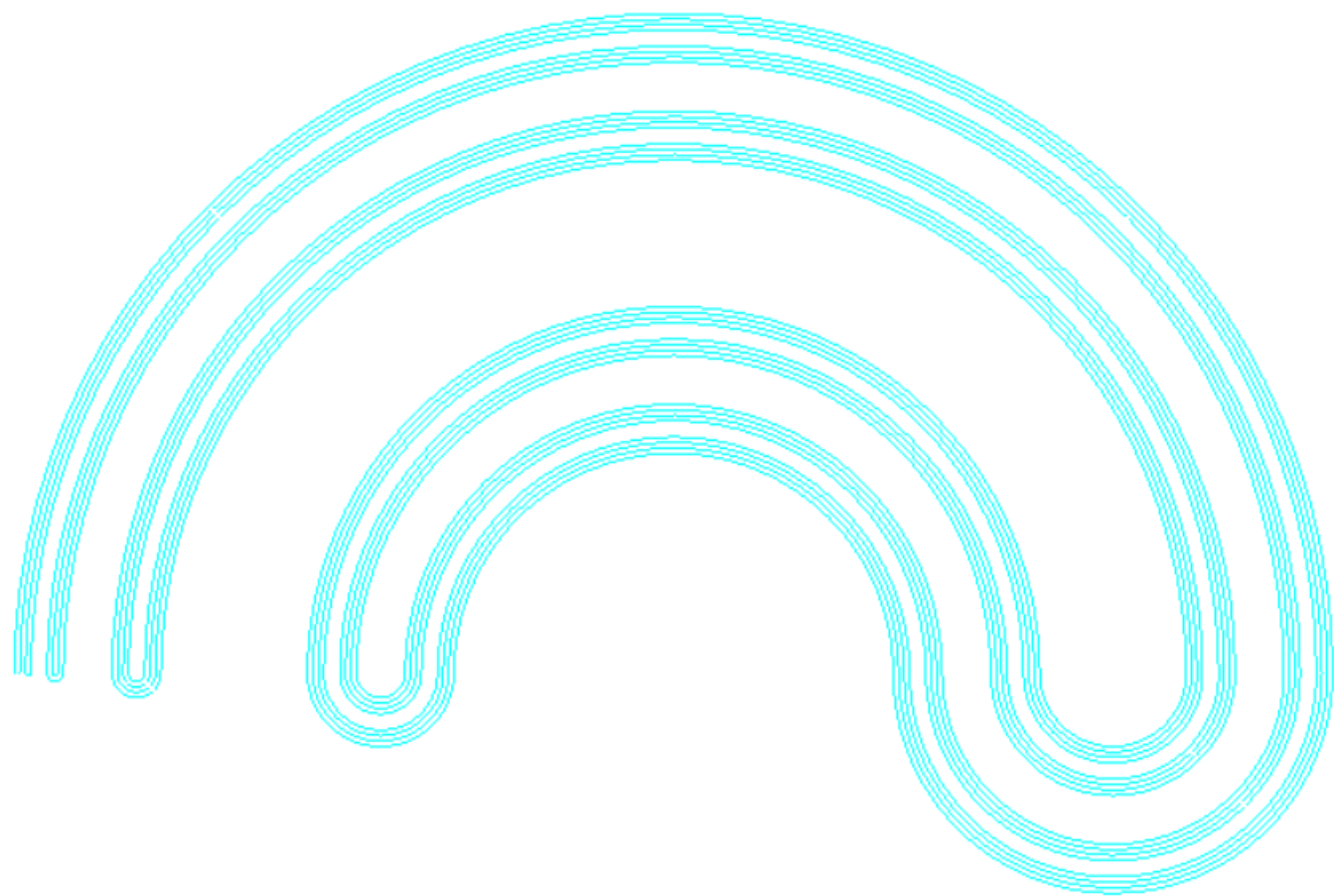# Continuum Theory Gem Award

- 1883-2020 setup was presented
- 1883-2021 update will follow ???

Proposal:

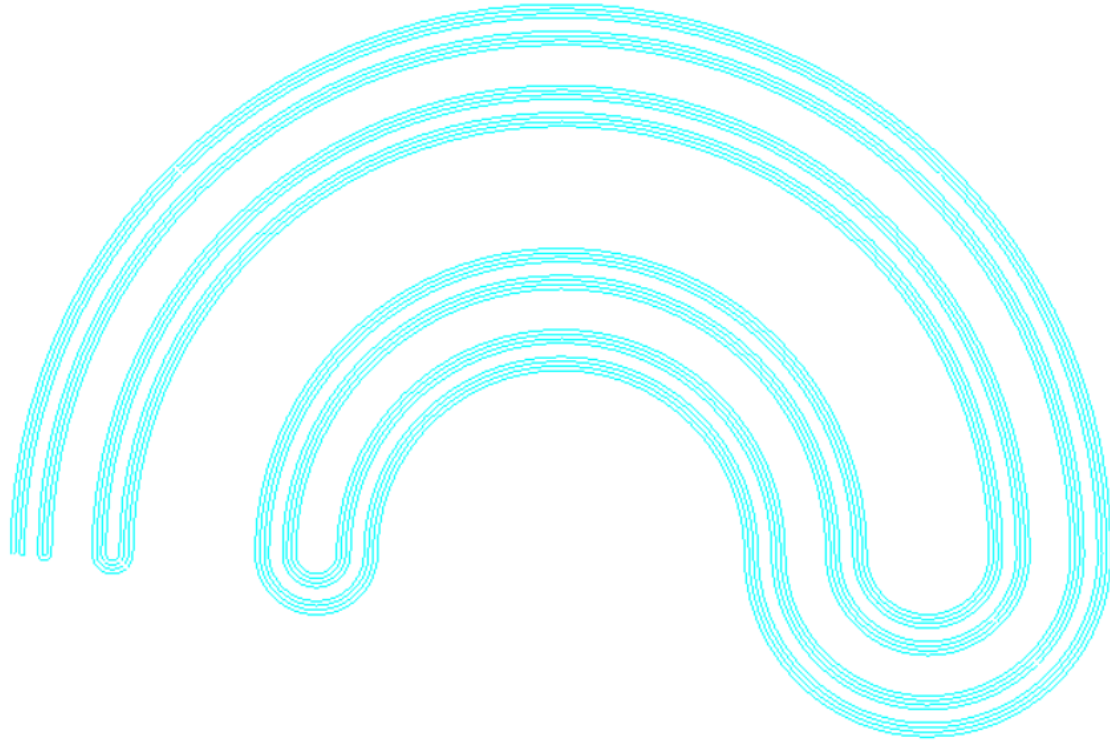Winners of the prize will update the list. Thanks.
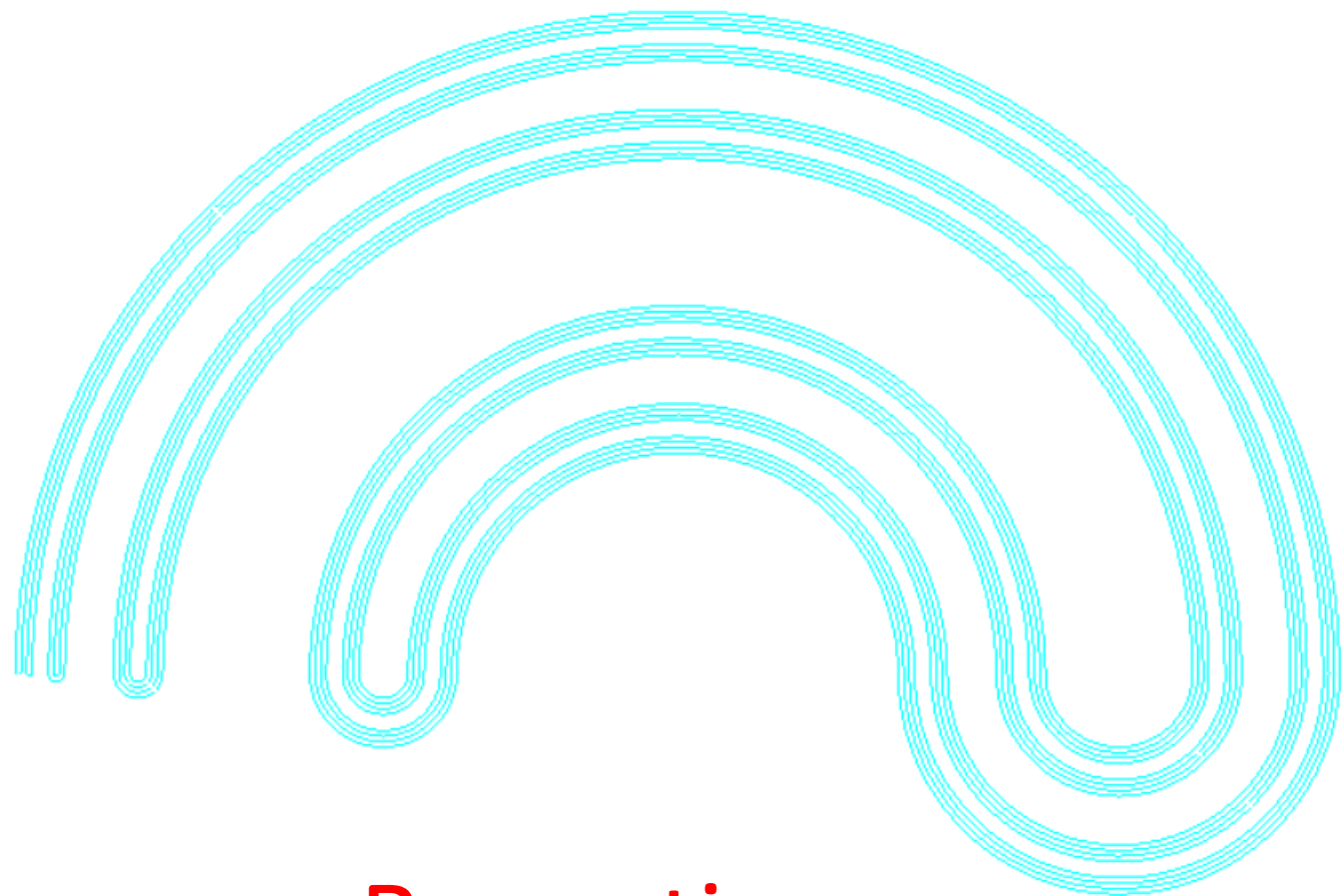
B₁ continuum
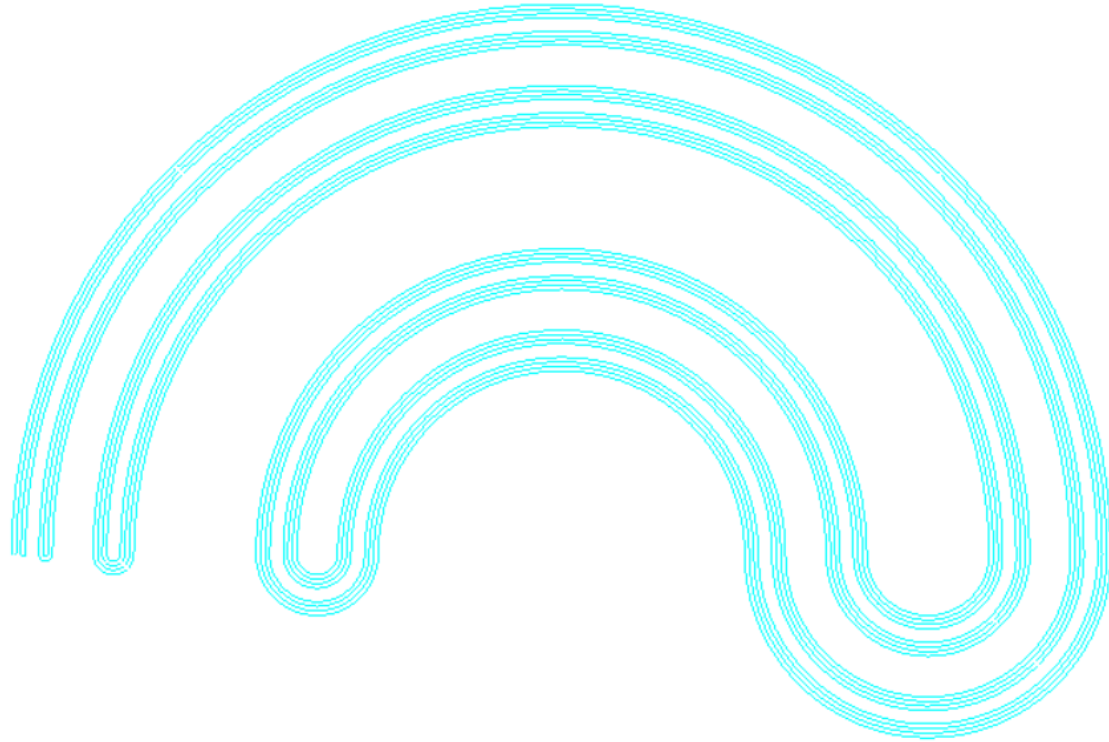
Motto: "Bee One Continuum"

math.cuni.cz

math.cuni.cz/dl/award

Media sources

- gem stones
- https://www.publicdomainpictures.net/en/view-image.php?image=4060&picture=gems
- diamonds
- https://www.publicdomainpictures.net/cs/free-download.php?image=velke-diamanty&id=205259
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- bag end
- https://pixels.com/featured/bag-end-hobbiton-new-zealand-neale-and-judith-clark.html
- wiki bag end
- https://en.wikipedia.org/wiki/Indecomposable_continuum#/media/File:ContinuBJK.svg